# Phased Array Calibration Procedures Based on Measured Element Patterns

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## ABSTRACT

A technique to compensate for differences in the element patterns of an array antenna is presented and discussed in comparison with previously published procedures. It follows the classical approach of correcting the array output vector by multiplication by a decoupling matrix to achieve a more or less unperturbed array response. The technique is applicable to linear as well as planar arrays and allows a rather flexible choice of the calibration points. The performance is demonstrated by a simulation based on a numerical analysis with the method of moments and a practical application to a Ku-Band digital beamforming array.

## INTRODUCTION

Optimal performance of a phased array antenna is only achievable by employing suitable calibration techniques to mitigate the non-uniformities in the patterns of the radiating elements. These are due to mutual coupling, differences in the amplitude and phase performance of the associated circuitry and near-field scattering. A convenient and suitable method to realise a correction of error effects is to apply a calibration matrix to the array output vector. This is done in order to obtain output signals as received by an ideal array system in the absence of mutual coupling and without any other error. Nevertheless, the determination of the matrix itself may require a great expenditure.

Different approaches to the problem of deriving this calibration matrix are known in literature. One way is to measure the scattering matrix of the antenna frontend and the complex transfer functions of the system channels. However, this may be rather impracticable for complex array systems or even impossible for integrated arrays, where the active circuits are an integral part of the array aperture. For this reason, several procedures calculate the calibration matrix from the measured patterns of array elements. In this paper, a very flexible technique of this kind is presented and discussed in comparison with procedures known from literature.

## CALIBRATION PROCEDURES

Based on the assumption of a single mode coupling between the antenna elements, a disturbed element pattern $g_n(u)$ of a linear array of $N$ elements with element separation $d$ can be modelled by the desired isolated element pattern $g^i(u)$ times a sum of the direct response and scattering from all other elements

$$g_n(u) = g^i(u) \sum_{m=1}^{N} c_{nm} e^{-jkmdu} , \tag{1}$$

where $k = 2\pi/\lambda$ is the wavenumber and $u = sin(\theta)$, with $\theta$ denoting the angle from broadside. The $c_{nm}$ represent the coefficients of a coupling matrix $\mathbf{C}$ describing the mutual coupling between the elements. By simply multiplying the array output vector $\mathbf{v} = \left[ \begin{array}{cccc} g_1(u) & g_2(u) & \cdots & g_N(u) \end{array} \right]^T$ by the inverse coupling matrix according to

$$\mathbf{v}^i = \left[ \begin{array}{cccc} g_1^i(u) & g_2^i(u) & \cdots & g_N^i(u) \end{array} \right]^T = \mathbf{C}^{-1}\mathbf{v} , \tag{2}$$

the pattern symmetry can, in theory, be restored completely. This, in effect, provides the signals as received by the desired patterns $g_n^i(u)$ referenced to the array phase centre. The function can be easily implemented in a digital beamformer and allows all subsequent beamforming or signal processing operations to be performed with ideal element signals.
The central problem of this calibration concept is the determination of the unknown matrix $\mathbf{C}^{-1}$.

### Classical Solutions

**Fourier Transform Technique.** The most popular technique to solve for the coefficients of the coupling matrix has been provided by Steyskal and Herd [1]. In their procedure, the $c_{nm}$ are derived as Fourier coefficients of the measured element patterns $g_n(u)$

$$c_{nm} = \frac{d}{\lambda} \int_{-\lambda/(2d)}^{\lambda/(2d)} \frac{g_n(u)}{g^i(u)} e^{-jkmdu}\, du \; . \tag{3}$$

This is of course the classical Fourier series technique used for shaped beam synthesis [2] applied to every element pattern of the array. It provides a least mean squared error approximation of each desired pattern for $d \geq \lambda/2$. For closer spacings, the domain of integration exceeds the visible region and the definition of the patterns is not unique.

**Beamspace Technique.** Instead of calculating integrals, the Woodward-Lawson synthesis technique derives excitation coefficients by superposition of sampling points of orthogonal array beams. For an array of $N$ elements, there are $N$ such beams at positions

$$u_i = (\lambda/(Nd))i \tag{4}$$

resulting from array illuminations with uniform amplitude and linear phase distributions [2]

$$e^{-jkndu_i} \; . \tag{5}$$

Aumann and Willwerth applied this method to the calibration problem [3] by describing each measured element pattern of the array $g_n(u)$ by a weighted sum of the orthogonal beams. Their interesting suggestion was an extension of the array beyond its physical size by virtual elements with the same separation, leading to more precisely sampled element patterns as well as a possibility to separate the mutual coupling from other disturbing effects.

For $M$ elements in the virtual array, the excitation coefficients are derived by

$$c_{nm} = \frac{1}{M} \sum_i \frac{A_{ni}}{g_n^i(u_i)} e^{-jkmdu_i} \; , \tag{6}$$

where, due to the orthogonality, the

$$(1/N)A_{ni} = g_n(u_i) \tag{7}$$

are the sample point values. The wanted coefficients of the coupling matrix are then given by an $N \times N$ sub-matrix of the coefficient matrix specified by (6). Actually, (6) is an extension to the procedure in [3], incorporating $g_n^i(u_i)$ into the denominator to account for directivity and phase centre of the desired pattern. Without this normalisation, the procedure may fail when applied to elements with a higher directivity or a phase centre that does not coincide with the aperture face.

A drawback to the technique is, that the sample points are determined by the method to be evenly distributed in $u$-space. Furthermore, it is not suitable for 2-dimensional arrays.

**Alternative Solution**

In order to overcome the limitations of the described procedures, but to still use a limited set of calibration points, an alternative is to select $M \geq N$ directions $u_m$ arbitrarily but sensibly distributed. Rewriting every row of (2) for every $u_m$ then results in an overdetermined set of linear equations

$$\begin{pmatrix} g_1(u_1) & g_2(u_1) & \cdots & g_N(u_1) \\ g_1(u_2) & g_2(u_2) & \cdots & g_N(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ g_1(u_M) & g_2(u_M) & \cdots & g_N(u_M) \end{pmatrix} \left(\mathbf{C}^{-1}\right)^T = \begin{pmatrix} g_1^i(u_1) & g_2^i(u_1) & \cdots & g_N^i(u_1) \\ g_1^i(u_2) & g_2^i(u_2) & \cdots & g_N^i(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ g_1^i(u_M) & g_2^i(u_M) & \cdots & g_N^i(u_M) \end{pmatrix} \tag{8}$$

with multiple right hand sides. With the more clearly rearrangement

$$\mathbf{G}\left(\mathbf{C}^{-1}\right)^T = \mathbf{G}\left[\begin{array}{cccc} \tilde{\mathbf{c}}_1 & \tilde{\mathbf{c}}_2 & \cdots & \tilde{\mathbf{c}}_N \end{array}\right] = \mathbf{G}^i \; , \tag{9}$$

the problem leads to an equation for unknown vector coefficients $\tilde{\mathbf{c}}_n$, representing the columns of the inverse and transposed coupling matrix.

Mathematically, the remaining task is to solve $N$ least squares problems

$$\min_{\tilde{\mathbf{c}}_n} \left\| \mathbf{G}\tilde{\mathbf{c}}_n - \mathbf{g}_n^i \right\|_2 \qquad n = 1, 2, \ldots, N \; . \tag{10}$$

A suitable method to do this is by QR factorisation [4] of the pattern matrix $\mathbf{G}$

$$\mathbf{G} = \mathbf{QR} \qquad \text{with} \qquad \mathbf{Q}^H\mathbf{Q} = \mathbf{E} \; , \tag{11}$$

where $\mathbf{Q}$ is a $M \times M$ matrix, $\mathbf{R}$ is upper triangular of dimension $M \times N$ and $\mathbf{E}$ denotes the identity matrix. The superscript $(\cdot)^H$ represents the Hermitian transpose (complex conjugate transpose). The $\tilde{\mathbf{c}}_n$ are then easily found by

$$\mathbf{R}\tilde{\mathbf{c}}_n = \mathbf{Q}^H\mathbf{g}_n^i \tag{12}$$

via backward substitution.

A significant advantage of this approach is, that it is very flexible in its choice of the calibration points. Furthermore, it directly calculates the coefficients of the wanted *de*coupling matrix, it has no restriction concerning the element separation and is also applicable to planar arrays.

## THEORETICAL INVESTIGATION

To assess the theoretical performance of the calibration techniques, an eight element linear aperture coupled stacked patch array (Figure 1) with element

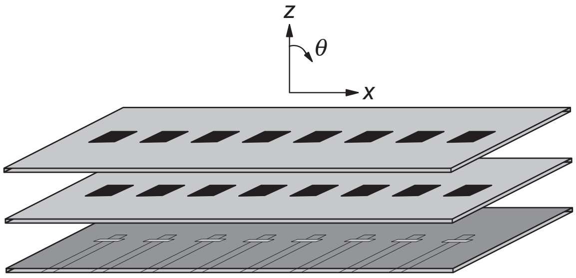


Figure 1: The investigated array of aperture coupled stacked patches

separation $d = 0.56\lambda$ was numerically modelled by the method of moments. All active element patterns as well as the isolated pattern were calculated and then used as data basis for the calibration procedures.

Figures 2 and 3 show the residual amplitude and phase deviations of the corrected element patterns $g_n^c(u)$ from the desired isolated element pattern

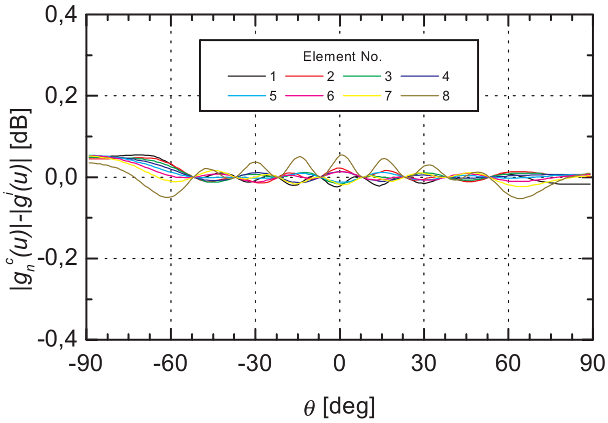


Figure 2: Theoretical amplitude deviations achieved by the new method

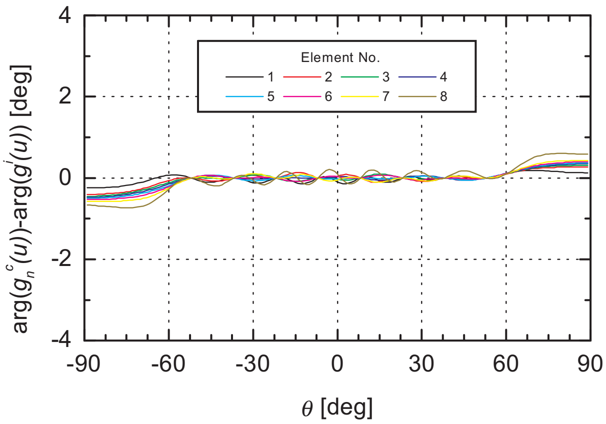


Figure 3: Theoretical phase deviations achieved by the new method

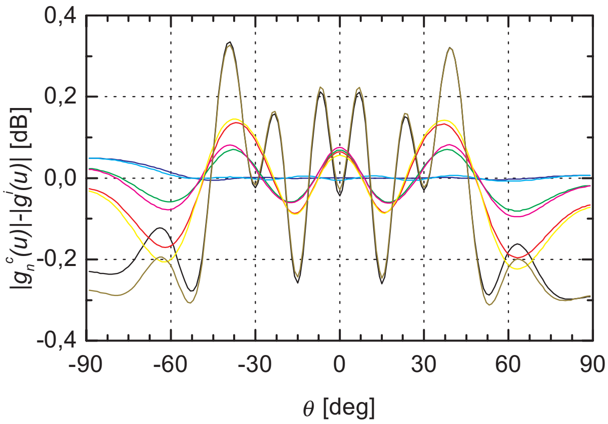


Figure 4: Amplitude deviations achieved by the modified procedure from [3]

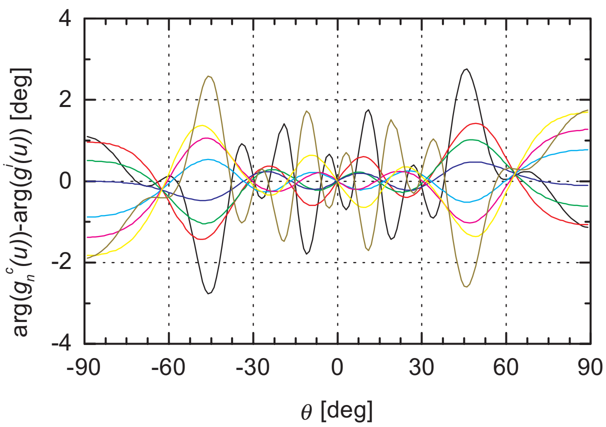


Figure 5: Phase deviations achieved by the modified procedure from [3]

$g^i(u)$. The procedure is in fact able to almost completely eliminate the pattern perturbations caused by mutual coupling.

In comparison, the calibration results obtained with the modified procedure from [3] are depicted in Figures 4 and 5. Again, a very good compensation of the errors is possible. For the examined array element type, this can only be accomplished by incorporating the newly introduced modification in (6), otherwise the procedure yields very unsatisfactory results. Nevertheless, the residual errors are larger compared to the new algorithm. This is based on the problem, that $u_i$-values determined by the positions of the orthogonal beams do not exactly match the calculated points of the patterns. However, a further improvement is possible by interpolation of the needed points.

The important conclusion from the theoretical analysis is, that both algorithms have the potential to effectively compensate for mutual coupling and amplitude and phase errors of the elements of an antenna array. The residual errors are due to higher order mode coupling in the antenna structure and cannot be completely corrected with the underlying data model. However, it can be shown by simulation, that the new calibration technique provides the exact solution for pattern imbalances based on single mode coupling between the antenna elements.

In real microstrip arrays, there exist several additional error effects besides mutual coupling, that cannot be easily accounted for during a numerical analysis. Especially, perturbations attributable to finite substrate dimensions and near field scattering cause an additional degradation of the element patterns and may result in severe problems for the data model in (1). It is then necessary to include more sample points into the calibration procedure to force a better averaging of the patterns.

## PRACTICAL IMPLEMENTATION

To assess the practical performance of the new algorithm, the calibration has also been applied to an active integrated eight element Ku-Band array operating at 12.7 GHz. It consists of the aperture coupled stacked patches that were used in the above simulations, integrated with eight separate channels for amplification and down-conversion to lower frequencies on the backside of the array board [5]. The rest of the array system realises a further down-conversion and a simultaneous sampling of all eight channels at a low IF frequency range. Finally, the array output vector is processed in a digital beamformer.

For the investigation, all element patterns of the antenna array were recorded in an anechoic chamber. Afterwards, suitable samples of the data were processed with the described algorithm to derive a decoupling matrix, that was then used for error correction during beamforming operations. Figure 6 shows the achieved array pattern for a $-35$ dB amplitude taper and a scan angle of $-30^{\circ}$ after a calibration with 32 calibration points distributed over visible space. The resulting pattern nearly exhibits the desired sidelobe level. Without the correction, the sidelobe control is precluded by the numerous error effects in the antenna frontend and the other parts of the system.

## CONCLUSION

An alternative solution to the problem of calibrating an array antenna has been presented and discussed in comparison with previously published techniques. The method is highly flexible in its choice of the calibration points and has no limitations imposed by the array geometry. The procedure has also been experimentally verified by application to a Ku-Band digital beamforming antenna system. The results show a substantial improvement in array system performance.

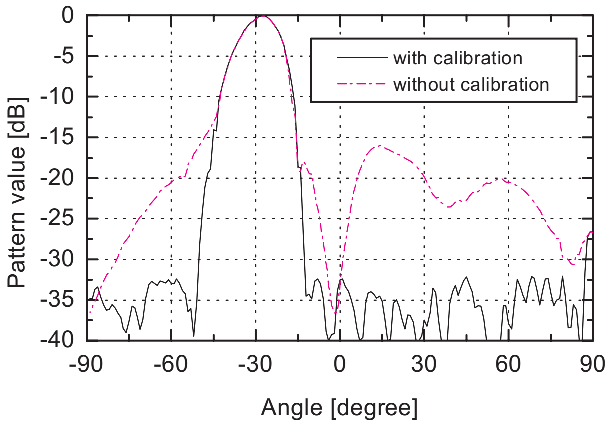


Figure 6: Scanned array pattern with sidelobe control before and after calibration with the presented method based on 32 sample points


## ACKNOWLEDGEMENT

The author is greatful to Prof. Dr. J. Marquardt for his support of this work and the helpful review of this paper.


## REFERENCES


[1] H. Steyskal and J. S. Herd, "Mutual coupling compensation in small arrays," *IEEE Trans. on Antennas and Propagation*, vol. AP-38, pp. 1971–1975, December 1990.

[2] W. L. Stutzman and G. A. Thiele, *Antenna Theory and Design.* John Wiley & Sons, 2nd ed., 1998.

[3] H. M. Aumann and F. G. Willwerth, "Phased array calibrations using measured element patterns," in *International Symposium on Antennas and Propagation*, (Long Beach, California), pp. 918–921, IEEE, June 1995.

[4] G. H. Golub and C. F. van Loan, *Matrix Computations.* The Johns Hopkins University Press, 3rd ed., 1996.

[5] L. Kuehnke, J. Marquardt, and J. Jaehrig, "An active integrated Ku-Band antenna breadboard for digital beamforming systems," in *Millennium Conference on Antennas and Propagation*, (Davos, Switzerland), April 2000.