Mutual Coupling of Open-Ended-Waveguides with Arbitrary Cross-Sections Located in an Infinite Groundplane

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***Abstract* - A Green's function approach is used to analyse mutual coupling in a finite array of open-ended waveguides located in an infinite groundplane. For the cross-sections of the cylindrical waveguides there are nearly no restrictions. In the waveguides the field is expressed as a sum of the transverse electric (TE) and transverse magnetic (TM) waveguide modes, and expressions for the mutual admittances of modes excited at the aperture are obtained using a direct integration approach. From these expressions the mode reflection and conversion coefficients are determined. Computed and measured results are presented for the reflection coefficient of the fundamental mode in a single waveguide due to the open end as well as for the mutual coupling between two waveguides for E- and H-plane dispositions.**

I. INTRODUCTION

Waveguides of circular, rectangular and even elliptical cross-section are commonly used as elements for direct radiating arrays and in array feeds for reflectors. The array radiation pattern, polarization properties and active impedance are all influenced by mutual coupling between elements [1, 2, 3].

This mutual coupling alters the mode content of the aperture distribution by causing the complex amplitude of the modes to differ from that of isolated elements, as well as sometimes generating other modes. For accurate prediction of the array performance this coupling should be included in any design procedure.

The mutual coupling in a finite array located in a ground plane can be analyzed very accurately using an integral equation and a Green's function approach [2, 3]. The integral equation is solved by replacing the fields in the apertures with a finite series of waveguide modes. The series coefficients are then determined by Galerkin's method.

In the past, only the cross-sections mentioned above were taken into account for determining the mutual coupling of waveguide elements [2, 3, 4]. This paper deals with a more general approach for the eigenmodes of the waveguides which allows to take into consideration nearly any kind of cross-section, for example a rectangular cross-section with rounded corners [5]. These roundings influence the cut-off frequencies and field distributions of all modes, which means that they influence the scattering parameters as well as the beam shape, the sidelobe level and the active impedance compared with a ideal rectangular cross-section.

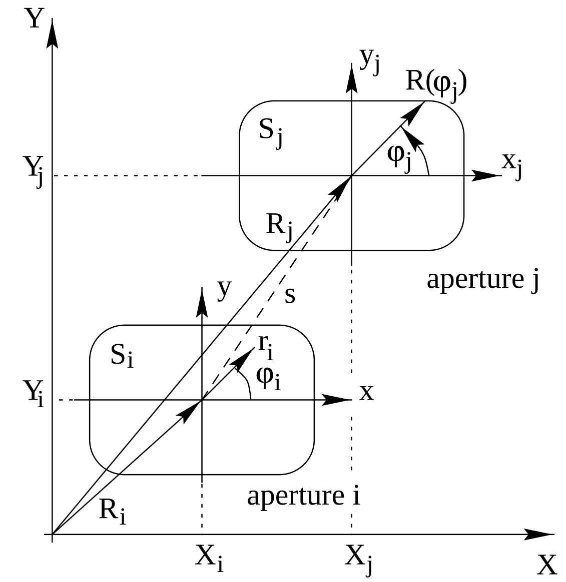

Fig. 1: GEOMETRY OF CYLINDRICAL WAVEGUIDES OPENING INTO A GROUNDPLANE

II. FORMULATION

A. Mutual Coupling

Consider N cylindrical waveguides terminating in a common ground plane as illustrated in Fig. 1. The field of each waveguide can be approximated as a sum of $M(i)\,(i=1...N)$ modes. In terms of the incident wave amplitudes at the apertures, the amplitudes of the reflected waves are

$$\vec{b} = \mathbf{S}\,\vec{a} \tag{1}$$

where

$$\mathbf{S} = 2(\mathbf{U}+\mathbf{Y})^{-1} - \mathbf{U} \tag{2}$$

is the modal scattering matrix of the complete array environment. $\boldsymbol{U}$ is the unit matrix and $\boldsymbol{Y}$ is the admittance matrix; $\vec{a}$ and $\vec{b}$ are the column vectors of the incident and reflected mode amplitudes. The elements of $\boldsymbol{Y}$ represent the mutual admittance of modes m and n in the apertures i and j, respectively. They can be calculated by the formula

$$y_{ij}(m,n) = \frac{jkY_0}{2\pi\sqrt{Y_m Y_n}} \iint_{S_i} \vec{\Psi}_m \cdot$$
$$\cdot \iint_{S_j} \vec{\Psi}_n G(x - x_j, y - y_j) dS_j dS_i \qquad (3)$$

where

Y_0 / Y_m = wave admittance in half-space / of mode m

k = wave number in half-space = $\omega\sqrt{\varepsilon_0/\mu_0}$

$\vec{\Psi}_m = \vec{h}_{tm} + k_{zm}/k \ h_{zm}\vec{e}_z$.

$\vec{h}_{tm}$, h_{zm} and k_{zm} are the transverse as well as the axial magnetic field and the wave number of mode *m*. $G(x, y) = \exp(-jkR)/R$ is the scalar Green's function with R = sqrt(x^2+y^2).

B. Eigenmodes of Waveguides with Arbitrary Cross-Sections

The computation of eqn. (3) requires the knowledge of the field distribution of the eigenmodes. This calculation is based on the expansion of the fields into basic solutions of the wave equation in polar-coordinates [5]. Therefore, the steady contour of each waveguide has to be formulated in these coordinates, as well. Thus, it is possible to describe a wide variety of waveguides, e.g. an elliptical waveguide or a rectangular waveguide with rounded corners as illustrated in Fig. 2. For the special case of a square waveguide, one can generate a circular cross-section (see Fig. 2d with $a = b = c$).

The normalized scalar potentials for waveguides with a π-periodic boundary and infinite conductivity have the following form for TE_m- (or H_m-) modes

$$F_{zm} = N_m \sum_{p=0}^{\infty} C_{pm} J_p(k_{cm}r)\cos(p\varphi - \psi_m) e^{-jk_{zm}z} \quad (4)$$

and for TM_m- (or E_m-) modes

$$A_{zm} = N_m \sum_{p=0}^{\infty} C_{pm} J_p(k_{cm}r)\sin(p\varphi - \psi_m) e^{-jk_{zm}z} \quad .(5)$$

Unlike the customary indication of the modes with two index numbers, only one index number *m* can be given here, since there is not a sole basic function per mode. An infinite number has to be assumed instead. That is why the modes have to be numbered all the way through their cut-off wavenumbers. The index number gives no clear indication of the appropriate field distribution.

With $k_{cm}^2 = k^2 - k_{zm}^2$, eqn. (4)-(5) satisfy the boundary conditions, and $\vec{\Psi}_m$ can be expressed in rectangular components for TE_m modes:

$$\Psi_{mx} = N_m \frac{k_{cm}}{2} \sum_{p=0}^{\infty} C_{pm} \cdot$$
$$\cdot[-J_{p-1}(k_{cm}r)\cos((p-1)\varphi - \psi_m)$$
$$+ J_{p+1}(k_{cm}r)\cos((p+1)\varphi - \psi_m)] \qquad (6)$$

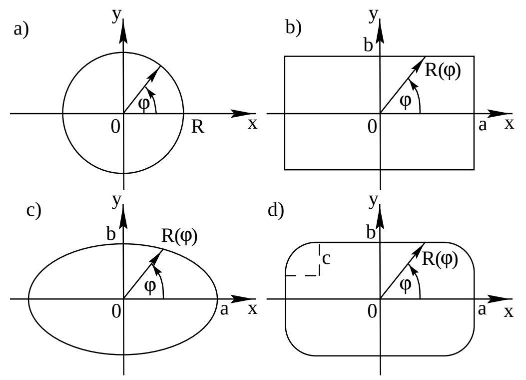

Fig. 2: GEOMETRY OF THE CROSS-SECTION OF DIFFERENT CYLINDRICAL WAVEGUIDES;
a) CIRCULAR b) RECTANGULAR c) ELLIPTICAL
d) RECTANGULAR WITH ROUNDED CORNERS

$$\Psi_{my} = N_m \frac{k_{cm}}{2} \sum_{p=0}^{\infty} C_{pm} \cdot$$
$$\cdot[\ J_{p-1}(k_{cm}r)\sin((p-1)\varphi - \psi_m)$$
$$+ J_{p+1}(k_{cm}r)\sin((p+1)\varphi - \psi_m)] \qquad (7)$$

$$\Psi_{mz} = N_m \frac{k_{cm}^2}{jk_{zm}} \sum_{p=0}^{\infty} C_{pm} \cdot$$
$$\cdot\ J_p(k_{cm}r)\cos(p\varphi - \psi_m) \qquad (8)$$

and for TM_m modes:

$$\vec{\Psi}_m^{TM} \Leftrightarrow \vec{e}_z \times \vec{\Psi}_m^{TE}$$
$$\Psi_{mz}^{TM} = 0 . \qquad (9)$$

In eqn. (4)-(9) N_m is the normalization constant *m*, C_{pm} are the expansion coefficients and k_{cm} is the cut-off wavenumber of mode *m*. The polarization angle ψ_m is defined relative to the initial line in the local polar-coordinate system (r, φ) that is parallel to the *y* axis for TE modes and parallel to the *x* axis for TM modes.

For a numerical computation the considered sum of the basic solutions of the wave equation in eqn. (4)-(5) has to be limited. As a rule, an upper limit of $p_{max} = 9$ is sufficient for most waveguide cross-sections. In Table 1, the cut-off wavenumber of the dominant TE-mode and TM-mode for a square waveguide (namely TE_{10} and TM_{01}) and a circular waveguide (namely TE_{11} and TM_{01}) are compared with the computed solution. As one can see, good agreement can be found. Further-more, the cut-off wavenumbers of waveguides with other cross-sections are also listed. These are generated by varying the radius of the rounded corners.

III. MODE COUPLING IN ARBITRARY SHAPED APERTURES

For modes coupling within the same aperture it is apparent from eqn. (3) that there is a singularity in the

Table 1: COMPARISON OF COMPUTED NORMALIZED CUTOFF WAVENUMBER $k_c b$ AND EXACT SOLUTION FOR b/a=1 (FOR a, b, c SEE Fig. 2d)

mode	exact solution for a square waveguide	computed solution c/b=0	c/b=0.25	c/b=0.5	c/b=0.75	c/b=1	exact solution for a circular waveguide
TE	1.570796	1.570801	1.591172	1.647432	1.731758	1.841184	1.841184
TM	2.221441	2.221499	2.222530	2.235281	2.285530	2.404825	2.404825

Green's function which must be treated very carefully. One way of doing this is to subtract the singularity out of the source region indicated in eqn. (10)

$$\begin{aligned} I &= \iint_{S_j} \vec{\Psi}_n \, G \, dS_j \\ &= \iint_{S_j} \vec{\Psi}_n \, [G - G_0] \, dS_j \\ &\quad + \iint_{S_j} \vec{\Psi}_n \, G_0 \, dS_j \end{aligned} \tag{10}$$

where $G_0 = 1/\mathrm{sqrt}[(x-x_j)^2+(y-y_j)^2]$ is the static field Green's function. For evaluating the second integral it is efficient to choose polar-coordinates (t, θ) with the origin at the field point (x, y) (see Fig. 3). After changing the variables to $x_j = x+t\cos\theta$ and $y_j = y+t\sin\theta$ this integral can be written as

$$I = \int_0^{2\pi} \int_0^{t(\theta)} \vec{\Psi}_n \, dt \, d\theta \tag{11}$$

where $t(\theta) = \mathrm{sqrt}[(R(\varphi_j)\cos\varphi_j - x)^2 + (R(\varphi_j)\sin\varphi_j - y)^2]$ is the corresponding upper limit of the radial integration. Since the contour of the aperture is known only by a pair of variables ($R(\varphi_j)$, φ_j) one has to express $t(\theta)$ in terms of $R(\varphi_j)$ and φ_j. This leads to

$$R(\varphi_j)\sin(\varphi_j - \theta) - r\sin(\pi - \varphi + \theta) = 0 \tag{12}$$

which has to be fulfilled by variation of φ_j. During the integration this problem occurs once for each integration angle θ. Though this procedure is valid for (x, y) inside the whole region of integration it is necessary to subdivide this area to achieve an unambiguous relationship between $t(\theta)$ and φ_j. The easiest way to do this is to deal with one quadrant after the other and to subdivide each quadrant into the regions defined by the dotted lines in Fig. 3.

Considering the case that the two apertures are separate no problem during a numerical integration occurs.

IV. RESULTS

The proposed method for evaluating the mutual admittances has been implemented in a program to compute the reflection coefficient as well as the coupling coefficients for modes in waveguides of rectangular cross-section with rounded corners. Fig. 4 shows the reflection coefficient of the dominant TE-mode in a square waveguide as a function of normalized frequency *kb* with the rounding factor *c/b* as a parameter. The results for $c/b = 0$ and $c/b = 1$, for square and circular cross-section respectively, can easily be verified by literature. A small radius of the rounded corners does not have a distinct influence on the reflection coefficient, while a larger radius has a great effect especially on the phase.

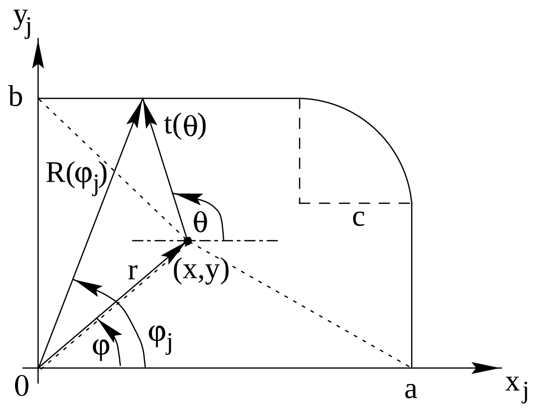

Fig. 3: COORDINATE-TRANSFORM FOR EVALUATING THE SINGULARITY

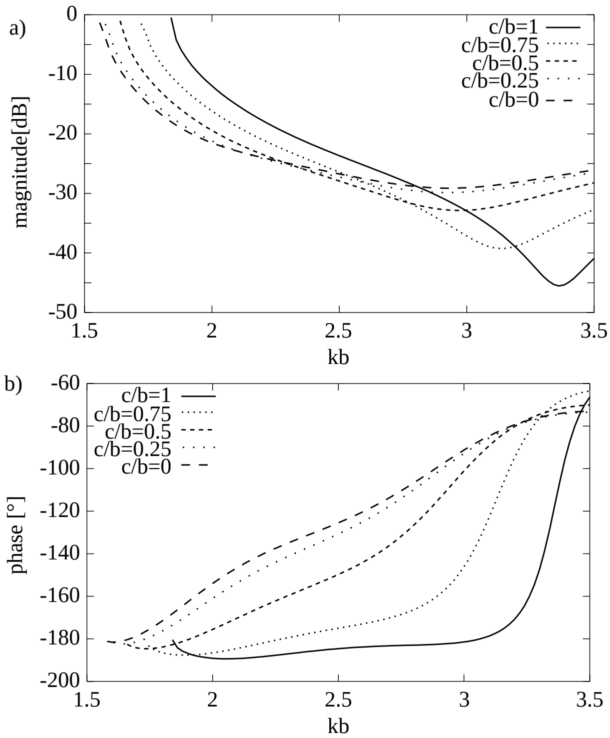

Fig. 4: REFLECTION COEFFICIENT OF TE_1 MODE AGAINST NORMALIZED FREQUENCY
a) MAGNITUDE b) PHASE

To prove the presented theory, the mismatch of a circular waveguide whose contour is built by a square waveguide with $a = b = 9.3$mm and $c/b = 1$ was measured

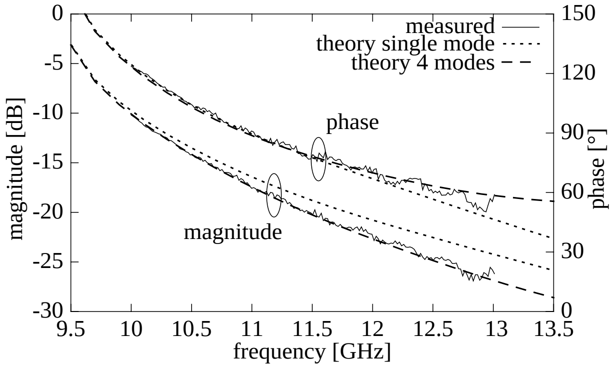

Fig. 5: REFLECTION COEFFICIENT AGAINST FREQUENCY

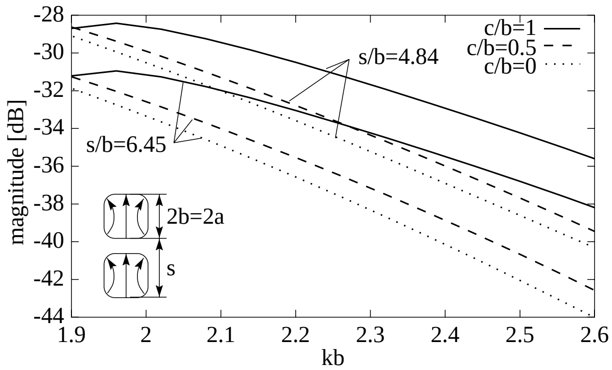

Fig. 6: E-PLANE COUPLING FOR IDENTICAL WAVEGUIDES AGAINST NORMALIZED FREQUENCY

using a conducting screen 6mm thick. The measurements are shown in Fig. 5 together with the computed values using a single mode and a four mode approximation for the aperture field. Though only a limited number of waveguide modes is used for representing the aperture field distribution, theory and experiment are in good agreement.

The coupling between two waveguides was calculated using four modes in each waveguide. Fig. 6 shows the results for $TE_1 \leftrightarrow TE_1$ coupling coefficient for identical waveguides located in the E-plane with a spacing s. The H-plane case is given in Fig. 7. As one can see, the parameter c has a great affect on the mutual coupling.

Measurements of the coupling coefficient were performed, as well. Two open-ended circular waveguides with $a=b$=9.3mm and c/b=1 were located in the conducting screen with a spacing s. The magnitude of the coupling coefficient of the dominant mode is shown in Fig. 8 for E-plane and H-plane dispositions. Once again, theory and experiment are in good agreement.

V. CONCLUSION

An analysis of mode coupling in an array of waveguides with nearly arbitrary cross-sections opening into an infinite ground plane has been presented. The formulation of this approach is valid not only for identical waveguides but may be applied to calculate

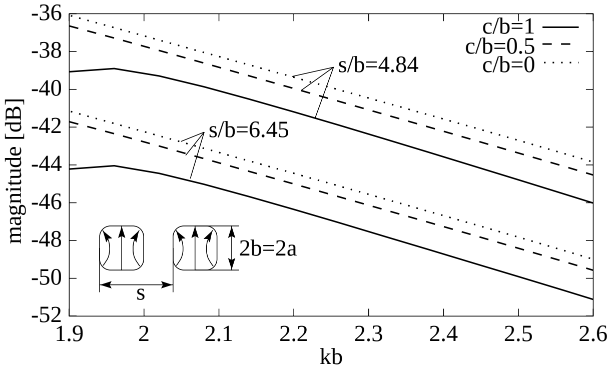

Fig. 7: H-PLANE COUPLING FOR IDENTICAL WAVEGUIDES AGAINST NORMALIZED FREQUENCY

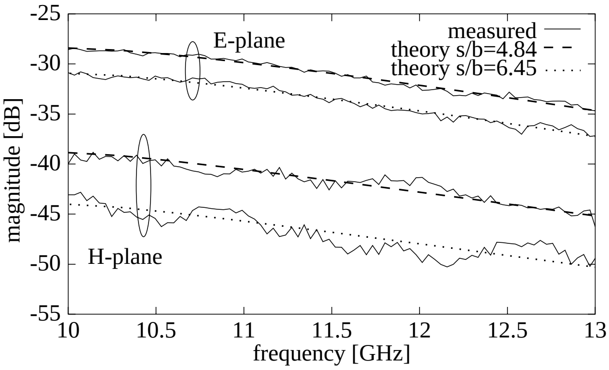

Fig. 8: MAGNITUDE OF COUPLING COUEFFICIENT FOR SQUARE WAVEGUIDES WITH c/b=1 AND SPACING s AGAINST FREQUENCY

coupling between dissimilar apertures, as well as to deal with different cross-sections in the same waveguide array. Theoretical results have been provided for square waveguides with the radius of the rounded corners as a parameter. The theory has been verified by measurements with circular waveguides. It has been observed that in the operating region of the fundamental mode the rounded corners do have a great effect on both reflection and coupling coefficients.

VI. REFERENCES

[1] A. Rudge, K. Milne, A.D. Olver, P. Knight, *The handbook of antenna design*. Vol.2. London, U.K., Peter Petegrinus, 1983.

[2] M.C. Bailey, „Analysis of finite-size phased arrays of circular waveguide elements“, *Technical Report R-408*, NASA, 1972.

[3] T.S. Bird, D.G. Batemann, „Mutual coupling between rotated horns in a ground plane“, *IEEE Transaction on Antennas and Propagation* **42, No. 7** (1994), 1000-1006.

[4] T.S. Bird, „Behaviour of multiple elliptical waveguides opening into a ground plane“, *IEE Proceedings, Part H* **137, No. 2** (1990), 121-126.

[5] J. Marquardt, M. Schneider, „Waveguide circuits with nearly arbitrary cross-sections“, *International Conference on Microwave and Millimeter Wave Technology Proceedings*, IMCMMT´98, Beijing, China (1998), 557-560.