COUPLING-EFFECTS IN FINITE ARRAYS OF OPEN-ENDED WAVEGUIDES WITH ARBITRARY CROSS-SECTIONS

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A new approach is given to analyse the mutual coupling of open-ended waveguides with arbitrary cross-sections located in a conducting screen. The field inside each waveguide is expressed as a sum of the transverse electric (TE) and transverse magnetic (TM) modes, and expressions for the mutual admittances of modes excited at the aperture are obtained using a direct integration method. From these expressions the mode reflection and conversion coefficients are determined. Computed and measured results are presented for the mutual coupling between two waveguides for E- and H-plane dispositions.

1 Introduction

Waveguides of circular, rectangular and elliptical cross-section are commonly used as elements for direct radiating arrays and in array feeds for reflectors. The array radiation pattern, polarization properties and active impedance are all influenced by mutual coupling between the elements [1, 2, 3]. For accurate prediction of the array performance this coupling should be included in any design procedure. The mutual coupling in a finite array located in a ground plane can be analysed very accurately using an integral equation and a Green's function approach. The integral equation is solved by replacing the fields in the apertures with a finite series of waveguide modes. The series coefficients are then determined by Galerkin's method. In the past, only the cross-sections mentioned above were taken into account to design waveguide arrays [1, 2, 3]. This paper deals with a more general approach for the eigenmodes of the waveguides which allows to take into consideration nearly any kind of cross-section, for example a rectangular cross-section with rounded corners [4]. One major reason for employing these elements in waveguide arrays is design flexibility.

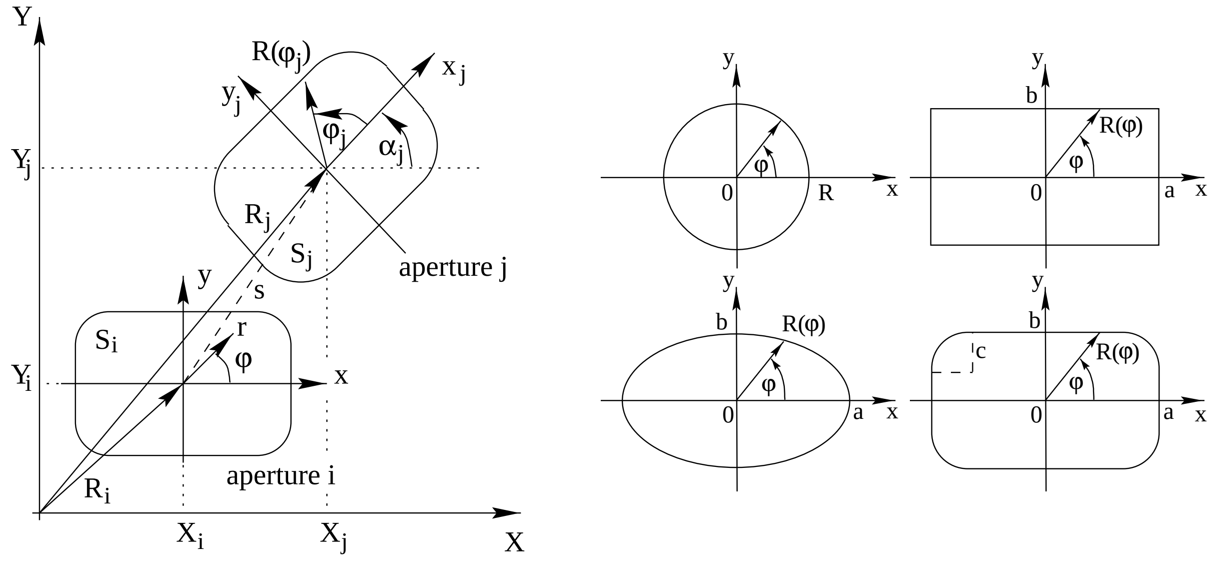

Figure 1: Geometry of cylindrical waveguides opening into a ground plane

Consider N cylindrical waveguides terminating in a common ground plane as illustrated in

Table 1: Comparision of computed normalized cut-off wavenumber $k_c b$ and exact solution for $b/a = 1$ (for a, b, c see Fig. 1)

mode	$k_c b$ for square waveguide	computed solution					$k_c b$ for circular waveguide
		$c/b = 0$	$c/b = 0.25$	$c/b = 0.5$	$c/b = 0.75$	$c/b = 1$	
TE_1	1.570796	1.570801	1.591172	1.647432	1.731758	1.841184	1.841184
TE_6	4.712390	4.712408	4.758722	4.849694	5.032082	5.331443	5.331443
TE_{12}	8.458997	8.459705	7.841794	7.854073	8.037948	8.536316	8.536316
TM_2	3.512407	3.512415	3.515110	3.542753	3.635977	3.831706	3.831706
TM_6	6.476560	6.476587	6.481088	6.523265	6.668454	7.015587	7.015587
TM_{13}	9.554781	9.555559	9.560036	9.594425	9.733982	10.17347	10.17347

Fig. 1. The field of each waveguide can be approximated as a sum of $M(i)(i = 1 \ldots N)$ modes. In terms of the incident wave amplitudes at the apertures the amplitudes of the reflected waves are

$$\boldsymbol{b} = \boldsymbol{S}\boldsymbol{a} \qquad \text{where} \qquad \boldsymbol{S} = 2\,(\boldsymbol{U} + \boldsymbol{Y})^{-1} \Leftrightarrow \boldsymbol{U} \tag{1}$$

is the modal scattering matrix of the complete array environment, $\boldsymbol{U}$is the unit matrix and $\boldsymbol{Y}$is the admittance matrix; $\boldsymbol{a}$ and $\boldsymbol{b}$ are the column vectors of the incident and reflected mode amplitudes. The elements of $\boldsymbol{Y}$represent the mutual admittances of modes m and n in apertures i and j, respectively and can be calculated by

$$y_{ij}(m,n) = \frac{jkY_0}{2\pi\sqrt{Y_m Y_n}} \int\!\!\int_{S_i} \boldsymbol{\Psi}_m \int\!\!\int_{S_j} \boldsymbol{\Psi}_n\, G(x \Leftrightarrow x_j, y \Leftrightarrow y_j)\, dS_j\, dS_i \tag{2}$$

with Y_0 as the wave admittance in free-space, Y_m as the wave admittance of waveguide mode m, k as the wave number in free-space and $\boldsymbol{\Psi}_m = \boldsymbol{h}_{tm} + k_{zm}/k \;\; h_{zm}\, \boldsymbol{e}_z$. $\boldsymbol{h}_{tm}$ and h_{zm} are the transverse and axial magnetic fields of mode m, k_{zm} is the wave number of mode m and $G(x,y) = exp(\Leftrightarrow jkR)/R$ is the scalar Green's function.

The computation of Eqn. (2) requires the knowledge of the field distribution of the eigenmodes of the waveguides. The calculation of the eigenmodes is based on the expansion of the fields into basic solutions of the wave equation in polar-coordinates [4]. Therefore, the steady contour of each waveguide has to be formulated in polar-coordinates (see Fig. 1). The normalized scalar potentials for waveguides with a π-periodic boundary and infinite conductivity have the following form:

$$\text{TE}_m\text{-(or H}_m\text{-) modes:} \qquad F_{zm} = N_m \sum_{p=0}^{\infty} C_{pm} J_p(k_{cm} r) \cos(p\varphi \Leftrightarrow \psi_m) e^{-jk_{zm} z} \tag{3}$$

$$\text{TM}_m\text{-(or E}_m\text{-) modes:} \qquad A_{zm} = N_m \sum_{p=0}^{\infty} C_{pm} J_p(k_{cm} r) \sin(p\varphi \Leftrightarrow \psi_m) e^{-jk_{zm} z}. \tag{4}$$

With $k_{cm}^2 = k^2 \Leftrightarrow k_{zm}^2$ Eqn. (3)-(4) satisfy the boundary conditions, and $\boldsymbol{\Psi}_m$ can be expressed in rectangular components:

$$\Psi_{mx}^{TE} = N_m \frac{k_{cm}}{2} \sum_{p=0}^{\infty} C_{pm} \left[\begin{array}{l} [\Leftrightarrow J_{p-1} \cos_{p-1} + J_{p+1} \cos_{p+1}] \cos\alpha_i \\ \Leftrightarrow [\;\; J_{p-1} \sin_{p-1} + J_{p+1} \sin_{p+1}] \sin\alpha_i \end{array} \right] \tag{5}$$

$$\Psi_{my}^{TE} = N_m \frac{k_{cm}}{2} \sum_{p=0}^{\infty} C_{pm} \left[\begin{array}{l} [\Leftrightarrow J_{p-1} \cos_{p-1} + J_{p+1} \cos_{p+1}] \cos\alpha_i \\ +[\;\; J_{p+1} \sin_{p-1} + J_{p+1} \sin_{p+1}] \sin\alpha_i \end{array} \right] \tag{6}$$

$$\Psi_{mz}^{TE} = N_m \frac{k_{cm}^2}{jk_{zm}} \sum_{p=0}^{\infty} C_{pm} J_p \cos_p \tag{7}$$

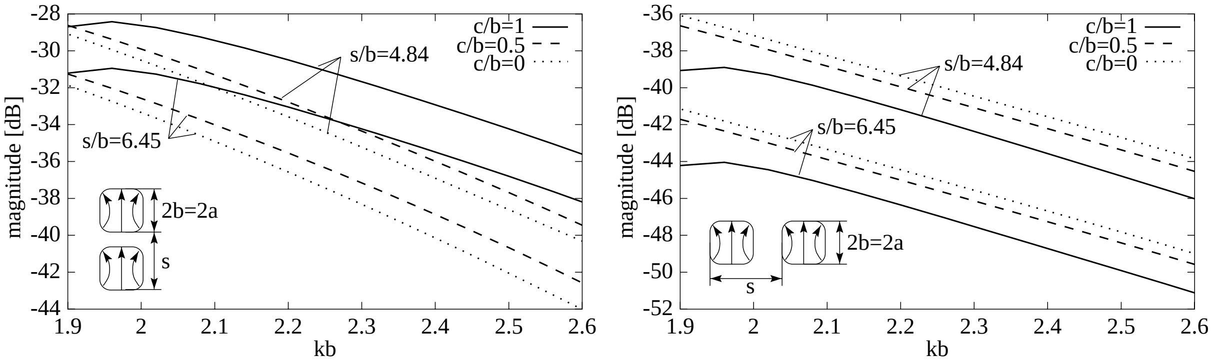

Figure 2: E-Plane and H-plane coupling for identical waveguides against normalized frequency

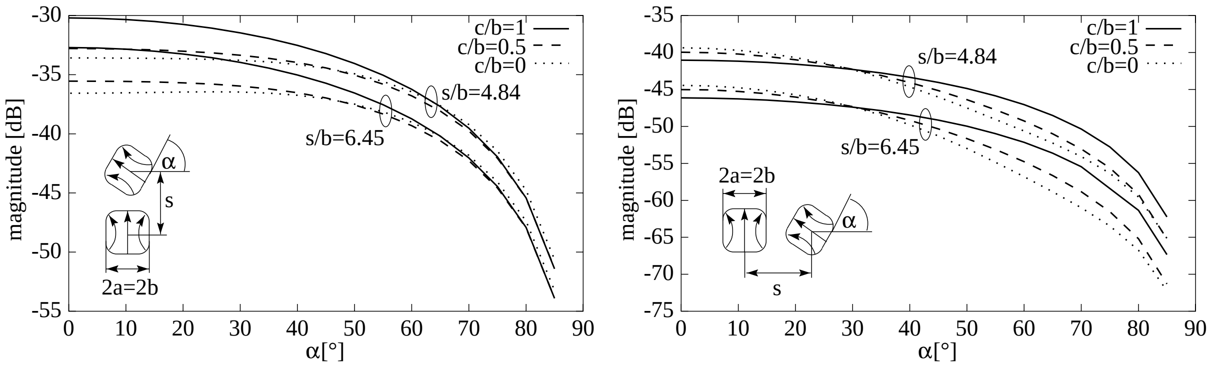

Figure 3: E-Plane and H-plane coupling for identical waveguides against rotation angle α at $kb = 2.2$

with $J_p = J_p(k_{cm}r)$ and $\cos_p = \cos(p\varphi \Leftrightarrow \psi_m)$ and

$$\mathbf{\Psi}_m^{TM} \Leftrightarrow \boldsymbol{e}_z \times \mathbf{\Psi}_m^{TE} \qquad \Psi_{mz}^{TM} = 0 \tag{8}$$

In Eqn. (3)-(8) N_m is the normalization constant of mode m, C_{pm} are the expansion coefficients, J_p is a Bessel function of order p, and k_{cm} is the cutoff wavenumber of mode m. The polarization angle ψ_m is defined relative to the initial line in the local polar-coordinate system (r, φ) that is parallel to the y axis for TE-modes and parallel to the x axis for TM-modes. The rotation angle α_i describes the orientation of waveguide i in the ground plane. Unlike the customary indication of the modes with two index numbers, only one index number m can be given here, since there is not a sole basic function per mode but instead a infinite number has to be assumed. Therefore, the modes are numbered all the way through their cut-off wavenumbers. For a numerical computation it is evident that the considered sum of the basic solutions of the wave equation in Eqn. (3)-(4) has to be limited. As a rule, an upper limit of $p_{max} = 9$ and $p_{max} = 15$ is sufficient to calculate the cut-off wavenumber of the dominant mode and higher order modes for most waveguide cross-sections, respectively. In Table 1, the cutoff wavenumbers of several TE- and TM-modes for a square and a circular waveguide as well as for a square waveguide with rounded corners are listed. Note that for a circular cross-section only one basic solution of the wave equation has to be considered.

The elements of the admittance matrix have to be evaluated numerically due to the arbitrary cross-sections of the waveguides. For modes coupling within the same aperture it is apparent from Eqn. (2) that there is a singularity in the Green's function which must be treated very carefully for accurate results. One way of doing this is to substract the singularity out of the source region as described in [3]. After changing the variables to $x_j = x + t\cos\theta$ and $y_j = y + t\sin\theta$ and taking care of the interdependence of the variables t, θ, φ_j and $R(\varphi_j)$, this procedure leads to an equation of the form $R(\varphi_j)\sin(\varphi_j \Leftrightarrow \theta) \Leftrightarrow r\sin(\pi \Leftrightarrow \varphi + \theta) = 0$ which has to be fulfilled by variation of φ_j at each integration angle θ. Eqn. (2) may then be integrated

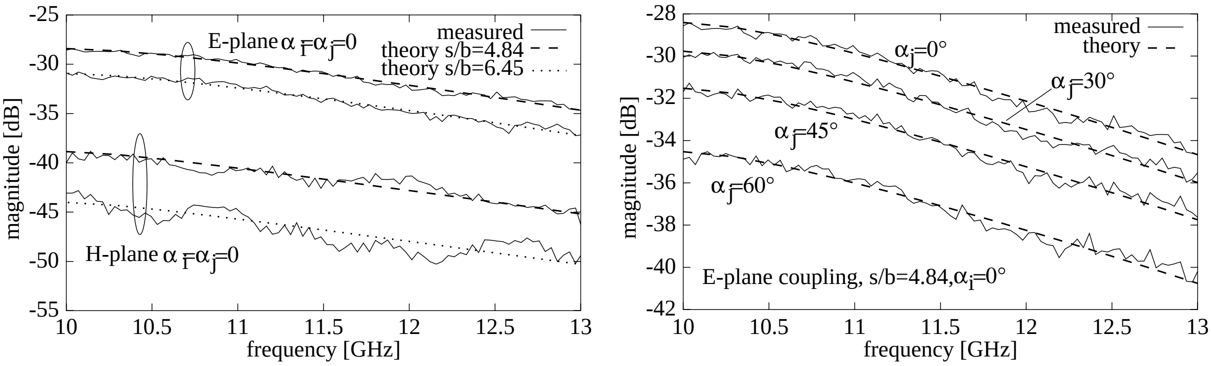

Figure 4: Coupling coefficient for waveguides with a=b=c (≡ circular cross-section)

numerically. For modes coupling within different apertures no problem during the integration occurs.

2 Results

The presented formulation was applied to calculate the coupling-effects between two identical waveguides. Fig. 2 (using four relevant modes) and Fig. 3 (using eight relevant modes) show the results for the coupling coefficient of the dominant mode for waveguides located in the E-plane and in the H-plane with a spacing s and c as a parameter, respectively. The presentation of the reflection coefficients is neglected. It can be observed that the radius of the rounded corners has a great effect on the mutual coupling.

Measurements were performed using circular waveguides, whose contours are built through $a = b = c = 9.3mm$, opening into a conducting screen of the dimension 250mmx300mm. Microwave absorbing material was attached to the edges of the screen in order to reduce edge diffraction effects. The results are shown in Fig. 4 together with the computed values using a four and a eight mode approximation of the aperture fields, respectively. Note, even though no averaging was performed, theory and experiment are in exellent agreement.

3 Conclusion

An analysis of mode coupling in a finite array of waveguides with arbitrary cross-sections opening into a ground plane has been presented. The theory has been verified through measurements, and it has been observed that rounded corners in rectangular waveguides have a great effect on the coupling coefficients. This may support the flexibility of an array design procedure. The presented formulation may also be applied to dissimilar apertures and waveguides with different cross-sections within the same array.

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